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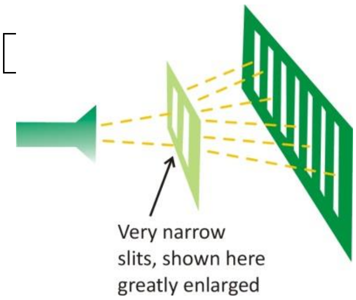
Learning station IV: Wave Particle Duality

1 Wave Particle Duality: fundamental to light and matter

In the 19th century it became clear that light was a wave phenomenon: a wave of **electric and magnetic fields**. But soon after it turned out that this did not exactly reveal the true nature of light.

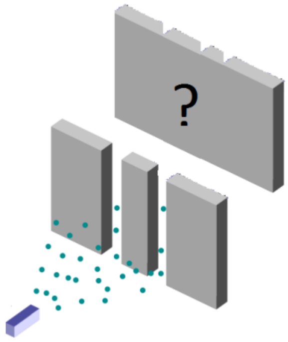


What would you expect to see on the screen if light arrives as particles? ....

What would you expect to see if dimmed light arrives as waves?

……………………………………………………………………

In the double slit experiment we saw that light can't be just waves nor particles: it does **not** arrive as **particles** (which would yield two bands) and **not** as **waves** either (which would produce an interference pattern that fades away): the interference pattern is made up particle by particle (learning station I). *It looks like the light particles agree how they will arrive: in an interference pattern of waves*. Thus, the light seems to be somewhat of a particle and a wave at the same time.



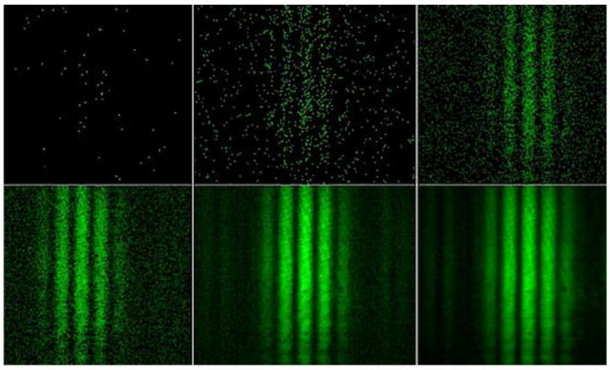


Figure 1 Light in a double slit experiment: it arrives as individual photons but they pile up to form an interference pattern as if they were waves (Recording by A. Weis, University of Fribourg)

The same seems to go for matter: electrons, neutrons and even molecules.

Can we perceive electrons or light in the double slit experiment as merely consisting of little balls? (YES/NO)

Can you compare the results of the double slit experiment with electrons to that of light? Is it comparable or not?

We thought before that at least matter would be matter-like, but even small matter particles do not arrive as separate bands after the slits. They built up an interference pattern like waves. So matter seems to have wave-like properties too.

Physicists call this unimaginable property of nature, the wave-particle duality. The strange nature of light is clearly not consistent with the classical view of being either particle or wave.

Wave-particle duality is a fundamental principle of light and matter

In nature, there is apparently a kind of symmetry between light and matter.

Electrons and photons in the 2-slit experiment arrive *one at a time*,
but the *pattern formed* by these particles is an interference pattern,
caused by the *wave* characteristics of these particles!

This wave-particle duality seems to be a fundamental principle in nature. Quantum mechanics has really changed our way of thinking about the nature of the world.

Here you see how a photo is built up also photon by photon (Source: Rose, A (1973) Vision: human and electronic. Plenum Press)

From the double slit experiment for light of low intensity, it is evident that *light* shows wave and .... properties.
From the double slit experiment for electrons, it is evident that *electrons* show particle and .... properties.

The particle model and wave model are complementary, and the reality is more complex than either of the two models can describe on its own. **Wave-particle duality** is a new fundamental characteristic of light and matter in physics. It is a characteristic brought by modern physics as opposed to classical physics.

2 Quantum Theory of light and matter

Einstein supposed that the **energy of light** can only be 'exchanged in small packets or **quanta'**. So not continuously as you would expect from waves. In other words the energy of the electromagnetic field cannot vary continuously, but only in small **discrete jumps**, called ***quanta***. The energy quanta (or the particles if you like) of the electromagnetic field are called **photons**.

2.a Electromagnetic waves and their energy quanta: photons.

Max Planck discovered the precise relationship between the energy of a photon, and the frequency of the electromagnetic wave. The famous Planck-Einstein relation gives the precise size of every energy packetof the electromagnetic field for every frequency of that field:

$$E = h.f \qquad (1.)$$

where h is a very small but fundamental **constant of nature** that indicates the fine granularity of light in quanta: the value of the **Planck's constant h= 6,63 × 10^{-34} J . s**

a) Calculate the value of the quantum of light for yellow light which has a frequency of f =5 x 10^{14} Hz

E = ...

This is the smallest quantum of energy that can occur in nature for light of this frequency.

b) Calculate the energy of three yellow photons:

E = ...

The energy of yellow light cannot take any arbitrary value but is always a (integer / not integer) multiple of the photon energy that you calculated above in a). **The energy of light is not continuously variable, but is quantized.**

But because light consists of an electromagnetic vibrating field, we conclude that the **energy of the electromagnetic field** itself is **quantized**. The energy can be issued only by the field per photon, thus in packets.

Experiment: in the hands-on experiments that go with these learning stations there is an experiment where you can determine Planck's constant by measuring the different voltages at which red, yellow, green and blue LEDs light up.

Technology: The fact that light is detected as energy quanta is used in every digital camera. In learning station VI you can learn about that application.

2.b Matter waves and quanta

Ok, the light particles we see are in fact energy packets of the electromagnetic field. But does the symmetry between light and matter goes so far that matter particles can be considered as the quanta of some sort of field too?

Years before the double slit experiment for electrons was performed, the French physicist **Louis de Broglie** predicted the *wave properties* of matter. Louis De Broglie believed in the symmetry of nature. Because electromagnetic waves interact in discrete quanta of energy (which we perceive as photons), it was clear to him that the opposite would also hold: particles of matter, up till then not considered to be the quanta of a field, should be viewed in this manner as well.

So De Broglie *postulated* the existence of matter waves. Thus he was also the first to propose the wave nature as a fundamental property for matter. Moreover De Broglie could write down a precise relationship that connected this matter field to the normal properties of matter like mass and velocity.

In this expression, De Broglie establishes a link between mass and velocity of a particle, and wavelength.

The product of mass and velocity is called linear momentum and is noted as 'p'. So

$$p = mv \qquad (2.)$$

Momentum is a classical **property of particles** also known in Newtonian mechanics (e.g. a truck at low speed can have a momentum as large as a small car at high speed).

On the other hand a **property of waves** is the wavelength. De Broglie now connects these previously incompatible properties of particles and waves in a precise relation:

wavelength
is a property of
waves

$$\lambda = \frac{h}{mv} \qquad (3.)$$

Linear momentum
is a property of particles.

Do you notice that in De Broglie's relation the particle and wave nature of matter are precisely connected through the tiny constant of Planck which acts as a proportionality constant?

The wavelength of (a quantum of) matter is inversely proportional to its linear momentum.

Particles that have a large amount of momentum, therefore, have a (LONG/SHORT) wavelength.

For an electron the de Broglie relation states: p = h / λ, where p is .... of the electron, and λ is the........................ that is connected to the electron.

How is it that we can **observe** (e.g. in the double slit experiment) the **wavelength of an electron** and that **we don't notice the wavelength of a ball**? Let us calculate the wavelengths of the two particles with the De Broglie relation.

a) Calculate the De Broglie wavelength of an electron with a speed of $v=6 \times 10^6$ m/s.

(You need the mass of the electron. Go look it up!) m_{e-}=………………………kg

λ= … ……………

b) Calculate the De Broglie wavelength of a ball with a mass m=0,20 kg and a speed v=15 m/s.

λ= …… ……………………

c) Compare these two wavelengths with the wavelength of light (look back to one of the previous learning stations where we talked about the electromagnetic spectrum):

The wavelength of visible light ranges between ………………

The wavelength of a normal ball is ………………than the wavelength of visible light.

The wavelength of an electron is ……………… than the wavelength of visible light.

d) What is the reason why we don't observe the wave characteristics of a normal ball (not even in a sophisticated lab)?

But what kind of a wave is a matter wave? Let's try to find an answer.

3 Quantum Fields

3.a Intensity of the waves gives the probability to dectect quanta

Let us look to the double slit experiment again, this time with the hypothesis of De Broglie that a field is connected even to matter.

How can we explain the interference pattern? The waves of the field connected to the electrons or neutrons can pass through both slits and these waves can indeed interfere with themselves. Thus, the superposition of both waves (one from each slit) is responsible for the creation of an interference pattern of maxima and minima. We know already that maxima occur on places where the path length difference is such that the wave originating from one slit is in phase with the wave coming from the other slit (constructive superposition). In other places, the path length is such that destructive interference occurs between the two waves.

Figure 2 Quantum Theory assumes that a field is connected to particles like an electron. The waves of that field pass through the slits and interfere (constructively and destructively) in different places. Where the amplitudes are large there is a large chance that a discrete and local energy packet is given off by the field. That is what we observe as a particle.

But the new insight is that where the maxima of the waves occur, it doesn't mean there will be a particle detected for sure. In places *where the amplitude of the field is maximal,* i.e. in the interference maxima, there is just a *high* ***probability*** **that an energy packet is released by the field**. Thus, in case of the electron double slit experiment, on those maxima there is a high probability that an electron emerges.

The intensity of the field gives
the probability that a quantum of the field can be detected.

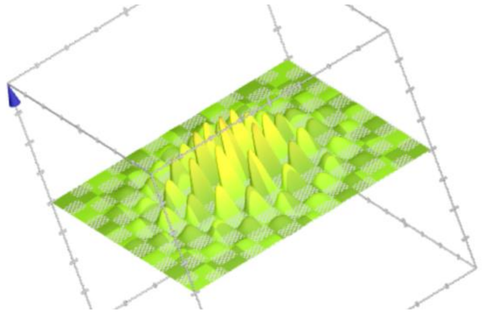
This fundamental property of ***probability*** *explains the* ***random*** *appearance of the spots ('particles') in the double slit experiments. The precise appearance of an electron cannot be predicted, only the chances.*

In the case of a double slit experiment for light, the same physical meaning holds for the electromagnetic field: the maximum number of photons occur on places where the intensity of the field is maximal: on those places there is a high probability that a photon can be issued by the field.

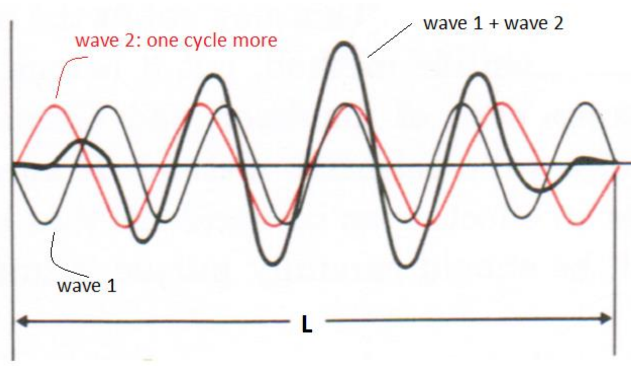
It was the German physicist Max Born who came up with this 'probability interpretation' of the intensity of the field. This might sound quite spooky and at first many physicists didn't believe it could be true, but it turned out to be the way nature works. That's why quantum waves are called 'probability waves'. It is the probability that varies, the probability that a quantum of energy (which we perceive as a particle) emerges from the field.

3.b A particle as a wave packet

A consequence of the wave nature of matter is that particles are not fully localised as they are in classical physics. In Newtonian mechanics a particle has a determined position and velocity. But due to the wave character of particles, the location and velocity is more 'smeared' out. Let us see what this means exactly.



In Quantum Theory you should think of a particle as a **wave packet** like you see in the figure above (or 'on the left' or right, depending on where you place the picture). A wave packet consists of a sum of different waves that constructively superimpose in a certain region. Further away the different waves that sum up, have phase differences that cancel each other out. As a result the particle has a probability of being detected only in a limited space.

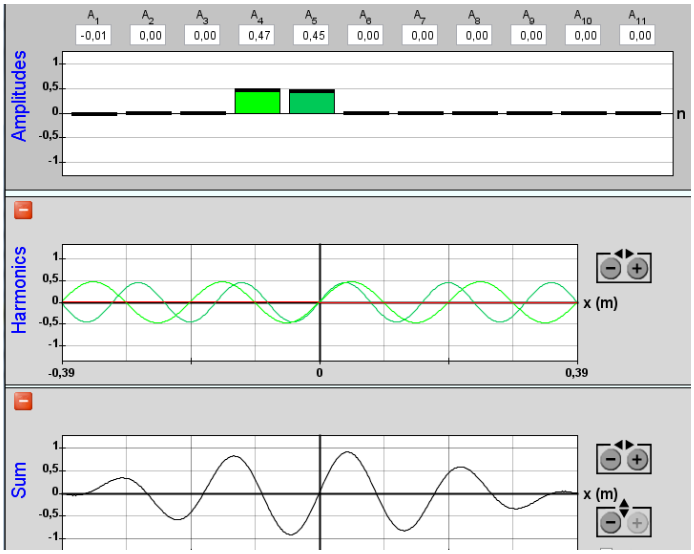


Look at this example: two waves sum up to form a third wave which is the wave packet. Here the 'view' is limited to 2 dimensions.
Do you see that the wave packet is maximal in the middle and fades away at the edges? It is all done by adding just two waves with the same constant amplitude: wave 1 has a wavelength that fits 4 times in the length L. Wave 2 has one cycle more so its wavelength fits …………… times in the length L. Do you see that the two waves show (constructive/destructive) interference at the two edges? In the middle they interfere (constructlively/destructively).

Exercise with the Phet applet

You can further examine how waves sum up to a wave packet in the Phet applet 'Fourier': https://phet.colorado.edu/en/simulation/fourier

Try to obtain the above situation with two waves that differ one cycle over the length L.



What possibility did you choose?

Number of cycles of wave 1: ………………….

Number of cycles of wave 2: ………………….

Now try to make the particle more localised. Can you do it by adding more waves?

By adding more waves with slightly a different number of cycles, there is more and more (constructive/destructive) interference further away. Only in the middle the waves sum up (constructively/destructively).

In fact the French mathematician Fourier proved you need to add an infinite number of waves with slightly different numbers of cycles.

Possible packets that could arise further away are 'killed' by adding more properly adjusted waves. So only the central packet remains. That's where the highest probability is for measuring a particle. However, there is still some probability of finding the particle around that central area, where the sum of the waves is not zero. So can we say with certainty where the particle is? ……………………………………………………………

Look back to the example: what can we say about the position of the particle, based on these considerations about probability? In other words, what is the "*indeterminacy*" or uncertainty of the position of the particle? ....

Before making a measurement, the particle is said to be in a "superposition state": this is a consequence of the wave nature. For example, when we perform a double slit experiment, we cannot know through which slit the particle will go, i.e. we do not know the position of the particle. However, when it interfers with the screen behind the double slit, then it appears in a precise position: we have made a measurement and the "superposition state" is gone away, i.e. the particle is in a well determined state as a result of the measurement.

3.c The Heisenberg indeterminacy principle

i) Indeterminacy in a particle's position and momentum

So a particle is not fully localised before it is detected, and there is no way of predicting 'where the particle actually is' like we are used to in classical physics. The consequence of this is that, unlike In Newtonian mechanics, we cannot determine the position and velocity of a particle at the same time. There is kind of a trade off: the more you know about the particles position, the less can be known about its velocity. Werner Heisenberg revealed that there is a precise indeterminacy relation between position and momentum (which is, as you recall the product of …………… with ………………). It is not a question of a bad measurement procedure that you can improve. It is an intrinsic property of particles: you cannot determine their position precisely because they have no precise position before a measurement. Now let us see how this is simply a consequence of the nature of the particle as a wave packet.

Reasoning for the particular case of two waves differing one cycle

Let us take show, at least for the case of the superposition of two waves that differ one cycle (over the length L), how this leads to indeterminacy

The number of cycles can be found by dividing the length L by the wavelength λ. So,

$$\frac{L}{\lambda} = number\ of\ cycles\ in\ length\ L$$

Because the difference in cycles in this particular case is 1, we can write that the difference

$$number\ of\ cycles\ of\ wave\ 2\ in\ length\ L - \ number\ of\ cycles\ of\ wave\ 1\ in\ length\ L = \ \dots\dots\dots\dots ?$$

Or in symbols:

$$\frac{L}{\lambda_2} - \frac{L}{\lambda_1} = \cdots$$

Now we bring in the De Broglie relation which is: ………………………….

and replace $\frac{1}{\lambda_n}$ by $\frac{p}{h}$

So we get: $\frac{Lp_2}{h} - \frac{Lp_1}{h} = \cdots \dots$

Or:

$$L(p_2-p_1) = \dots\dots\dots$$

Now, looking again at the example above, L is the indeterminacy of the wave packet (or the particle if you like), which is commonly noted as Δx.

And (p_2-p_1) is the indeterminacy of the momentum (mass times velocity) of the particle, mostly noted as Δp. This will indeed give the range of possible values (states) that the momentum of the wave packet can have.

So we get the following indeterminacy relation (check if your result is the same as below):

$$\Delta x \, . \, \Delta p = h$$

In our particular example of the superposition of 2 waves, we end up with a wave packet that is smeared out over L (or Δx). Due to the De Broglie relation where the wave is connected to a particle's momentum p, this implies a Δp too. So the product of the indeterminacies is at least h in this case. h is the minimum possible value for the case of adding the two waves. So we should write "≥"

$$\Delta x \, . \, \Delta p \geq h$$

General indeterminacy relation

A more general deduction than the above, points out that the product of the indeterminacy of position and momentum of a particle has to be larger than h/4π

$$\Delta x \, . \, \Delta p \geq h/4\pi$$

This is the famous **Heisenberg indeterminacy principle** which states that you cannot determine a particle's position and momentum deliberately accurate, not because of the shortcomings of your measuring instruments but simply because of the intrinsic wave character of particles. The particle does not possess an intrinsic accurate position and momentum. The indeterminacy of the product Δx . Δp is always larger than h/2π.

So quantum physics brought us the insight that an absolute determination is not a property of nature (which physicist believed until the advent of quantum physics). On the other hand nature's laws are not totally deliberate either. It is all more subtle and a lot of philosophical implications are still under discussion, about what this all means for our universe and the life it contains.

Exercise:

Let's calculate the indeterminacy in the velocity of an electron that is bound to an atom of 10^{-10} m diameter

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(Answer: $\Delta v \geq 10^6$ m/s !)

As you can see the question 'what is the velocity of the electron in the atom?' is quite meaningless if the velocity of the electron in the atom is not determined within such a large value. It shows again that the classical Rutherford picture where an electron orbits the nucleus is quite an insufficient scientific model.

ii) Indeterminacy in a particle's energy and time

There is also an indeterminacy trade-off between the energy and the time.

It is also a consequence of the wave character of particles. You can compare this to the situation where you want to determine the frequency of a tone. If you want to do that, you have to listen to the tone (or measure the tone if you like) for a certain time. So there is some relation

$$\Delta f \,.\, \Delta t \geq \text{some value}$$

Ok, this indeterminacy relation holds in classical physics like the physics of sound. You can't determine the frequency of a wave if the wave does not last for at least one cycle, in the case of one cycle $\Delta t=T$ and since $\Delta f= 1/T$ the 'some value' will equal 1.

$$\Delta f \,.\, \Delta t \geq 1$$

Now since particles in *quantum physics* have wave character too and the frequency of the quantum waves are related to the energy by the Einstein-Planck equation E=h.f, we get the following result, by replacing f by E/h

$$\frac{\Delta E}{h}.\Delta t \geq 1$$

And so we get the indeterminacy relation between the energy and the time:

$$\Delta E \,.\, \Delta t \geq h$$

The exact form is also in this case

$$\Delta E \,.\, \Delta t \geq h/2\pi$$

So a particle's energy is only defined within the limits of a certain time interval. You cannot speak of a certain energy of a particle *at a given time* like you can do in classical physics.

4 Quantum Field Theory

We saw earlier how scientists felt uncomfortable about the idea of 'action at distance' with respect to forces like gravitational or magnetic forces. To make this 'action at distance' scientifically sound physicists introduced the concept of **field**. Classical fields, such as the electric or gravitational **field**, are considered as "mediators" of forces (or rather energy). In quantum theory, the idea of field stays and also the notion that fields carry energy. Moreover, the idea of field is also connected to matter itself which is quite unexpected.

In fact, physicists developed quantum theory further in what is known today as **Quantum Field Theory**. **QFT** describes in principle everything we observe in the universe (or at least tries to do so): matter and forces. Compared to classical fields (like gravitational or the classical electromagnetic fields) *quantum fields* can only issue energy in discrete packets or "quanta".

Quantum fields are, like classical fields, defined in all points of space. They may vary over time as a wave, and in this way they can propagate.

For example, a **photon** may be created as the quantum of an **electromagnetic field**. But also matter itself, such as electrons and protons are quanta of matter fields. E.g. all particles measured at particle accelerators like CERN in Geneva, are predicted by quantum field theory too.

In the modern view of quantum field theory even forces (with remote action, remember!) are seen as the result of the exchange of energy quanta between fields.

But there are still some problems: the gravitational field still cannot be understood within the quantum field theory. Gravitation is therefore treated as a mere classical field, even in Einstein's general relativity. Physicists have been searching for about 100 years, for a quantum theory of gravity. Everyone hopes that the experimental verification of the Brout-Englert-Higgs boson in july 2012, will give some further outlook on that issue (cf. Nobel Prize in physics for Belgian François Englert and Scotsman Peter Higgs in 2013). New physics might be in sight when seeking for new answers to not yet understood phenomena such as **dark matter** and **dark energy**.

But only the future will tell to which new insights and ultimate applications this all will lead.

Figure 3 The Belgian François Englert and the Scottsman Peter Higss after the experimental confirmation of the Brout-Englert-Higgs boson in july 2012 at Cern. They both received nobel prize in December 2013 for physics, for the prediction of this quantum of the BEH-field, they made earlier in the 60'ies. (Source CERN, Geneva)

5 Concepts in Learning Station IV

Complete by adding the missing concepts

Classical concepts:

Momentum is a classical property of ………………………

Wavelength is a classical property of ………………………

The concept of field as "mediator" of forces.

Position and velocity of particles are ……………………………………

Quantum concepts:

The energy of the electromagnetic field can be issued only in ………………………... The energy of light cannot vary continuously, but only in ………………………………, called ………………...

Light has a wave-particle nature.

………………………………….. is a fundamental characteristic of light and matter.

The energy of the electromagnetic field itself is quantized. The size of the energy packages of this field (the photons) can be calculated with the Planck-Einstein relation: $E=h\cdot f$, where f is …………………………………..

De Broglie's hypothesis connects the particle and the wave nature of matter: $\lambda = h/mv$, where λ is ………………………………………………………. and mv the ……………………………………………………….

Quantum field theory: quantum fields can only issue energy in ………………………………………..

A photon may be created as the quantum of an …………………………………………………...

Matter, such as electrons and protons, are quanta of vibrating ………………………………………………...

The discrete interference pattern in the double slit experiment is in fact the image of the ……………………………………………… that an energy packet (quantum) is released by the (electromagnetic or matter) field.

Properties of particles, like position and velocity, are intrinsically ………………………………